FTD-ID(RS)T-1784-78

1

FOREIGN TECHNOLOGY DIVISION

TESLA TRANSFORMERS

By

Werner Heise

78 12 27 208

EDITED TRANSLATION

FTD-ID(RS)T-1784-78 17 October 1978

MICROFICHE NR: FTD-78-C-001407

TESLA TRANSFORMERS

By: Werner Heise

English pages: 24

Source: Elektrotechnische Zeitschrift,
Vol. 85, Nr. 1, 10 January 1964,
pp. 1-8

Country of Origin: East Germany
Translated by: LINGUISTIC SYSTEMS, INC.
F33657-76-D-0389
Chester E. Claff Jr.
Requester: FTD/TQTD
Approved for public release; distribution
unlimited.

NTIS	White Section ☒
DDC	Buff Section ☐
UNANNOUNCED	☐
JUSTIFICATION	
BY	
DISTRIBUTION/AVAILABILITY CODES	
Dist.	AVAIL. and/or SPECIAL
A	

PREPARED BY:

TRANSLATION DIVISION
FOREIGN TECHNOLOGY DIVISION
WP.AFB, OHIO.

Tesla Transformers

by: Werner Heise*)

The Tesla transformer, named after its inventor, is a resonance transformer. Transformers of this type of construction consist of two electrical oscillating circuits inductively coupled with one another, tuned to one another in their inherent frequencies. Free-oscillating resonating transformers which are generally called Tesla transformers, and resonance transformers with forced oscillation are differentiated. As is apparent from this terminology, the manner of excitation of the primary circuit characterizes a resonance transformer.

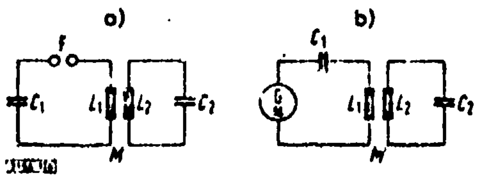

Figure 1. Circuit principles of resonance transformers.
a) Free-oscillating resonance transformer or Tesla transformer.
b) Resonance transformer with forced oscillation.

Figures 1a and 1b show the circuit principles of these two types of resonance transformers. In the first case, the oscillating system is excited with a surge by discharge of the previously charged condenser C_1 into the primary circuit, and in the second case, continuously with the help of a high frequency generator G operating at the resonant frequency of the circuit. Because of these differing manners of excitation, either a series of damped oscillations which can have heterodyne character under certain conditions, or an undamped oscillation, i.e., an oscillation

*) Dipl.-Ing. W. Heise is manager of the high voltage experimental Department of the AEG High Voltage Institute in Kassel.

with constant amplitude, will occur. With the help of resonance transformers, particularly with those of the free-oscillating type, voltages of the order of magnitude of 10^6 V can be generated without particular difficulty, if the individual elements of the circuit are appropriately designed.

Since the damping in the resonant circuits is a controlling factor for the voltage level that can be produced, resonant transformers are significantly more sensitive to any external electrical load than are transformers whose operating frequency is much lower than its resonant frequency. The lower the damping of the transformer itself, and the better the resonance conditions were met before application of a load, the more the switching on of a test circuit loaded with a specific impedance will affect the level of the voltage and its frequency. With respect to its sensitivity towards an external load, the resonance transformer is comparable with a 50 Hz test transformer which has a very large magnetic dispersion.

Tesla transformers are probably the best known representative of the group of resonance transformers. As already pointed out, the designation Tesla transformer today generally applies to all transformers of the free-oscillating type shown in Figure 1a.

Since Tesla transformers since the beginning of research in the field of high voltage technology, for several decades offered the only possibility of generating voltages in the order of magnitude of 100 kV and more, it is not surprising that papers appeared very early which were concerned with their theoretical study [1 to 4]. These papers treated among other things, questions of the effect of damping and of the degree of coupling on the attainable voltage levels and the voltage form that was set up. In the period between 1914 and 1932, on the

other hand, no further noteworthy publications concerning resonance transformers were published. This may have been because the theoretical treatment of the problem appeared to have been brought to a close, but that suitable means were not yet available for practical investigations, which would have provided further stimulation. Only the development of the cathode ray oscillograph permitted the examination of the theories developed up to that time relative to their agreement with actuality. Hochhausler, in the years between 1930 and 1932, carried out experimental studies on a Tesla transformer for a maximum voltage of approximately 10^6 V [6]. In his paper "The Tesla Transformer as a High Frequency Test Generator and Its Study With the Cathode Oscillograph", he indicated among other things, the good agreement between his calculations in the design and the measurement results in the testing of this Tesla transformer. The calculations were carried out essentially according to the data presented by Drude [1,2]. The studies of Hochhausler, however, also showed that the computational consideration of the damping effect caused difficulties.

Although Hochhausler at this time considered the Tesla transformer as a useful voltage source for testing high voltage equipment, particularly with respect to its electrical strength at high frequency overvoltages, this type of testing was not able to prevail because of the rapid development of the shock potential generator which was better suited for this purpose. On the other hand, however, high frequency high voltages were used for a long time in many fields of testing, to reveal concealed cracks and cavities in ceramic insulating materials. In subjecting such defective test pieces in this way to a voltage of high frequency, they heat up as a result of comparatively intense gas discharge currents in the enclosed cavity. This testing method, however, was also outdated by the modern process of ultrasonics testing technology.

These remained essentially the only disclosed technical applications for Tesla transformers, if such equipment with comparatively low voltage is disregarded such as is used even today, for example, for the testing of insulation and as a means of instruction in the schools. In recent years, even engineering schools and technical institutes have become interested in the construction of Tesla transformers for high voltages. It is possible that the Tesla transformer will also be shown in the future to be useful in assisting scientific studies.

THE THEORY OF THE TESLA TRANSFORMER

Figure 2 shows the electrical circuit diagram of a Tesla transformer. In this case, the circuit is excited by alternating voltage with a low frequency compared to the resonance frequency of the Tesla transformer. The primary condenser C_1 is charged up to the breakdown voltage U_0 of the spark gap F. The luminous arc which then occurs between the spark electrodes of F closes the primary circuit. The inductively coupled oscillatory arrangement consisting of the primary and secondary circuits is thereby stimulated and begins to oscillate. The voltage form and the frequency of this oscillation are dependent on the quality of the tuning, the degree of coupling, and the damping.

Figure 3 shows the fundamental curve of time with the voltage u_{C1} at the primary condenser C_1 and that of the secondary voltage u_2. At the time t_0, the charging of the condenser C_1 begins, for example by an alternating voltage whose time curve is indicated for the case of no load on the supply transformer. At time t_1 occurs the breakdown of the spark gap F already mentioned. Both circuits begin to oscillate. The frequency of these oscillations is $f = (f_I + f_{II})/2$, as will be derived later. At time t_2, the luminous arc at the generally air-ventilated circuit spark gap becomes extinguished, i.e., the oscillation in the primary circuit is interrupted and the condenser C_1 is again charged to

U_0 in the time (t_3-t_2) dependent on the time constant of the circuit and the level of the supply voltage. In the same interval of time, the secondary circuit decays alone at its own inheren frequency f_2, until it is again stimulated at time t_3 and repeats the illustrated cycle.

The equivalent circuit corresponding to Figure 2 is illustrated in Figure 4. With respect to its action on the processes taking place in the primary and secondary circuit of the Tesla transformer, the equivalent elements L_E, C_E, and R_E of the excitation circuit parallel to the primary condenser, can generally be neglected. The system of coupled differential equations valid for the inductively coupled circuits 1 and 2 then reads

$$\begin{aligned} \left(j\omega L_1 + \frac{1}{j\omega C_1} + R_1\right) I_1 + j\omega M I_2 &= 0, \\ \left(j\omega L_2 + \frac{1}{j\omega C_1} + R_2\right) I_2 + j\omega M I_1 &= 0. \end{aligned} \qquad (1)$$

The ohmic resistances R_1 and R_2 which are to be considered as the equivalent series resistors for all losses in circuits 1 and 2, will be neglected for the further treatment of the equation system (1) for reasons of clarity. The question of the damping will be considered in the next section.

With the simplifications

$$\frac{M^2}{L_1 L_2} = k^2, \quad \frac{1}{L_1 C_1} = \omega_1^2 \quad \text{and} \quad \frac{1}{L_2 C_2} = \omega_2^2$$

the characteristic equation of the system according to Equation (1), provided that the resonant frequencies of the two circuits coincide, and are equal to $\omega_1 = \omega_2 = \omega_0$, provides the two frequencies

$$\omega_I = \frac{\omega_0}{\sqrt{1-k}} \quad \text{and} \quad \omega_{II} = \frac{\omega_0}{\sqrt{1+k}} \tag{2}$$

for the coupled oscillations. Depending on the value of the coupling factor k, oscillations with the frequencies ω_I and ω_{II} are superimposed in the primary and the secondary circuits. The frequency of the resulting oscillation is $\omega = (\omega_I + \omega_{II})/2$. For coupling factors of 0.1 to 0.3 customary in practice, provided the primary circuit remains closed, beats with a beat frequency $\omega_s = \omega_I - \omega_{II}$ will form. For the theoretically possible limiting

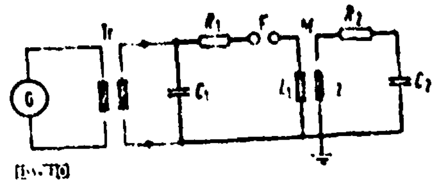

Figure 2. Circuit of a Tesla transformer with excitation device.

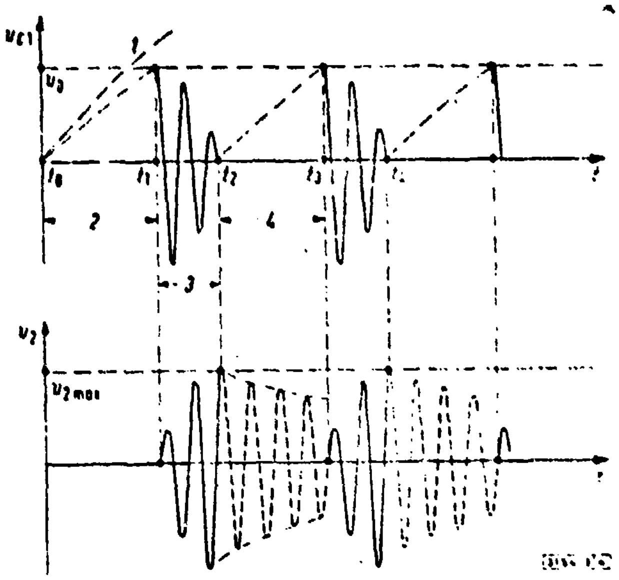

Figure 3. Fundamental time curves of the voltages u_{C1} and u_2 in a Tesla transformer whose condenser C_1 is charged with alternating voltage.
t_1, t_3 circuit spark gap ignites
t_2, t_4 circuit spark gap extinguishes
1 No-load voltage of the supply transformer
2 Charging period of C_1
3 Primary and secondary circuits oscillate at $t = (t_I + t_{II})/2$
4 Secondary circuit oscillates alone at $t_2 = t_0$

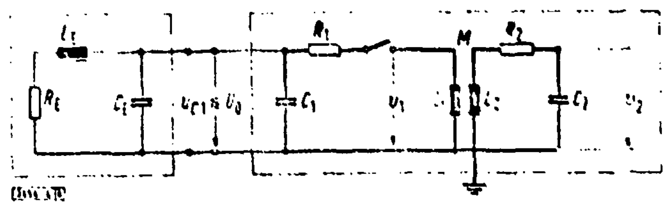

Figure 4. Equivalent circuit of a Tesla transformer with excitation device.

values of k, which are k = 0 and k = 1, however, only a single frequency appears in each case, namely $\omega = \omega_0$ and $\omega = \omega_0/\sqrt{2}$. Beats cannot occur in these cases. They are also absent at normal values of k, when a spark gap is used as a circuit element in the primary circuit, since the luminous arc in general already comes to an end at one of the zero current points of the first beat minimum. This is certainly true when the spark gap is cooled with the help of an intense stream of air, as is usually the case (Figure 3).

If the time curve of the primary voltage is designated as u_1 the secondary voltage as u_2, and the breakdown voltage of the spark gap F (Figure 2), or the voltage across the condenser C_1, at which another element of hte circuit closes, as U_0, then with the help of equations

$$\begin{aligned} u_1 &= A_1 \cdot \cos \omega_I t + B_1 \cdot \cos \omega_{II} t, \\ u_2 &= A_2 \cdot \cos \omega_I t + B_2 \cdot \cos \omega_{II} t \end{aligned} \qquad (3)$$

and the boundary conditions $u_1 = U_0$ and $u_2 = 0$ for the time $t = 0$, for the constants A_2 and B_2 of primary interest

$$A_2 = -B_2 = \frac{M C_1 U_0}{\sqrt{(L_1 C_1 - L_2 C_2)^2 + 4 k^2 L_1 C_1 L_2 C_2}}. \qquad (5)$$

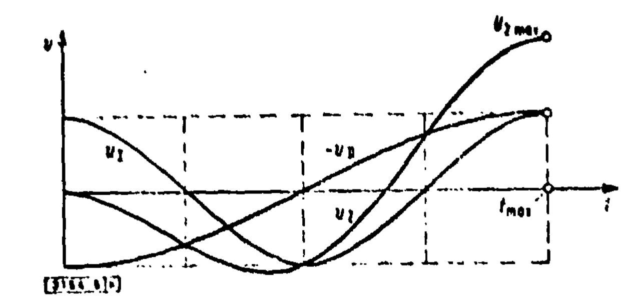

Figure 5. Time curve of the coupled oscillations u_I and u_{II} and the secondary voltage resulting from them u_2 for the case where the coupling factor k = 0.6.

This means that the amplitudes of the oscillations superimposed on the secondary side with the frequencies ω_I and ω_{II} are of equal size. By introducing the equation

$$\frac{L_2 C_2}{L_1 C_1} = x$$

Equation (5) can be transformed into

$$A_2 = -B_2 = \frac{1}{2}\sqrt{\frac{C_1}{C_2}}\, U_0 \frac{1}{\left[\frac{1-x}{2k\sqrt{x}}\right]^2 + 1}. \tag{6}$$

It can be seen from this illustration that the maximum values of A_2 and B_2 are reached only when $x = 1$, i.e. $\omega_1 = \omega_2 = \omega_0$

The relationship

$$A_2 = -B_2 = \frac{1}{2}\sqrt{\frac{C_1}{C_2}}\, U_0. \tag{7}$$

is then independent of the degree of coupling.

Without considering the effect of damping, the secondary voltage at resonance is therefore

$$u_2 = \frac{1}{2}\sqrt{\frac{C_1}{C_2}}\, U_0 \left[\cos\left(\frac{\omega_0}{\sqrt{1-k}}\, t\right) - \cos\left(\frac{\omega_0}{\sqrt{1+k}}\, t\right)\right]. \tag{8}$$

Because $\omega_1 = \omega_2 = \omega_0$, the factor $\sqrt{C_1/C_2}$ is also replaced in this equation by $\sqrt{L_2/L_1}$. The maximum voltage theoretically possible is therefore

$$u_{2\,max} = \sqrt{\frac{C_1}{C_2}}\, U_0 = \sqrt{\frac{L_2}{L_1}}\, U_0. \tag{9}$$

For the theoretical case of a loss-free transformer, u_{2max} results when the sum of the time functions from Equation (8) reaches the highest possible value of 2 at time t_{max}.

Actually, however, the attainable voltage can only more or less closely approach this value u_{2max} depending on the degree of the resulting damping. To keep the effect of damping on the maximum voltage small, it must be attempted to make the time t_{max} small. Since t_{max} can be set equal to half a period of the beat resulting from the coupled oscillations, then

$$t_{max} \approx \frac{1}{2} \cdot \frac{2\pi}{\omega_I - \omega_{II}}, \tag{10}$$

and since ω_I and ω_{II} from Equation (2) are dependent on the coupling factor $k = M/\sqrt{L_1 L_2}$, it can easily be seen that t_{max} becomes smaller with increasing k. Further considerations now show that it is advantageous to give the coupling factor k completely determined values if the conditions just specified are to be realized.

The first possibility of reaching the maximum of the secondary voltage apparently exists at time $t_{max} = T_{II}/2$, which amounts to half of the period of the slower coupled oscillation u_{II}, specifically when $T_I = T_{II}/2$, (Figure 5). In this case, therefore, it must be true that $\omega_I = 2\omega_{II}$ or $k = 0.6$, as can be calculated from

$$k = \frac{(\omega_I/\omega_{II})^2 - 1}{1 + (\omega_I/\omega_{II})^2} \tag{11}$$

As can easily be understood, for integral multiples of the ratio $\omega_I/\omega_{II} = 2$, the value u_{2max} is also obtained at time $t_{max} = T_{II}/2$. However, correspondingly larger coupling factors, according to Equation (11), are necessary for this. Nevertheless, no one will urge a greater expense in the construction of technical

equipment than is absolutely necessary to reach a specific goal, besides the fact that coupling factors of 0.6 for high voltage Tesla transformers are already practically out of reach. Therefore, the value $k = 0.6$ can be designated as a practical upper limit for the coupling factor.

Table 1. Desirable values for the coupling factor k

n	t_{max}	ω_I/ω_{II}	k
1	$1/2 \cdot T_{II}$	2	0,6
2	T_{II}	1,5	0,385
3	$3/2 \cdot T_{II}$	1,33	0,28
4	$2\,T_{II}$	1,25	0,222
5	$5/2 \cdot T_{II}$	1,2	0,18
6	$3\,T_{II}$	1,17	0,153
7	$7/2 \cdot T_{II}$	1,145	0,134

As was explained, the lower limit for the time t_{max} at which u_{2max} can be reached, is $t_{max} = T_{II}/2$. Other possibilities also exist for integral multiples of this time, i.e., for $t_{max} = n/2 \cdot T_{II}$. With the provision that the frequency ratio ω_I/ω_{II} and therefore the degree of coupling k is again chosen only large enough so that the amplitudes of the coupled oscillations at time t_{max} agree in size and sign, the equation

$$\frac{\omega_I}{\omega_{II}} = \frac{n+1}{n} \tag{12}$$

can be used. From this, together with Equation (2), there results for the coupling factor

$$k = \frac{\left(\frac{n+1}{n}\right)^2 - 1}{1 + \left(\frac{n+1}{n}\right)^2} \tag{13}$$

In this equation, n, as already stated, is a whole number. It is easily checked, that for n = 1, and therefore for $t_{max} = T_{II}/2$, the coupling factor k = 0.6. From Equation (13), therefore, those values of k are calculated which are to be used in consideration of the fact that the sum of the time functions according to Equation (8) approach the value 2 also for the case of infinitely large damping. In Table 1 are listed the desirable values for k and ω_I/ω_{II} for n.

Next, a fact important for computation of Tesla transformers should be pointed out, which has only been treated inadequately in the literature up to now. Because of the large geometric dimensions which transformers of this type have for high and very high voltages both for reasons of insulation technology and for purposes of a large transformer ratio, the wire length of its secondary coil is of the order of magnitude of the wave length of the voltage oscillations. The secondary coil must therefore be considered as a short conductor, and because of the ground capacities decreasing at the high voltage end, an inhomogeneous conductor, on which standing waves are formed. Now, if the maximum voltage u_{2max} should be tapped between the upper and lower ends of the winding, and if no other voltage maxima should be located along the winding for reasons of optimal utilization of the insulation then care must be taken that the wire length l of the secondary winding is equal to $\lambda/4$, where λ is the wave length of the oscillation. For the calculation of λ, the rate of propagation of the oscillation along a single-layer winding can be assumed to have a value of approximately $3 \cdot 10^5$ km/sec. If the condition $l = \lambda/4$ is then fulfilled, the distribution of the voltage with respect to ground along the winding is described approximately by the function

$$u_x = u_{2max} \sin\left(\frac{\pi}{2} \cdot \frac{x}{h}\right) \qquad (14)$$

where x is the distance of a point of the winding from its grounded end, and h is the overall height of the coil. This distribution was confirmed experimentally (Figure 6).

In order to be able to consider the condition $l = \lambda/4$ from the outset, the dimensioning of a Tesla transformer will have to begin with the calculation of the secondary circuit. For this, the symbol h is used for the height of the winding in cm, D for the average winding diameter in cm, and N as the number of turns.

If it is now assumed that the height of the winding h of the Tesla transformer corresponding to the desired highest secondary voltage is chosen only according to the technical requirements of insulation , then the maximum number of turns N_{max} is therewith also fixed, if the greatest winding density is considered to be fixed by the conductor cross section and the insulating coating. The wire length l of the winding is then dependent only on the average diameter of the winding D, which is $l = N\pi D$. However, this means also that the lowest possible value of the frequency f of the voltage is given by the requirement $\lambda/4 = l$, if a specific D has been decided upon, since h and N_{max} can only be selected with conditional freedom in accordance with the above statements. If a propagation rate of the secondary oscillation is assumed to be $v \approx c = 3 \cdot 10^{10}$ cm/sec., then there results for the lowest possible frequency, the fitted quantitative equation

$$\frac{f}{\mathrm{Hz}} \approx 0{,}75 \cdot 10^{10} \left(\frac{l}{\mathrm{cm}}\right)^{-1}. \qquad (15)$$

If, accordingly, values of h, N, and D have been settled upon, then there result automatically the values for the required resonance frequency, the inductance L_2, and the necessary capacity C_2 of the secondary circuit. For the size of the inductance L_2 of the cylindrical secondary coil generally made as a single-layer winding, the following equation applies to a good approximation, according to [8]:

$$L_2 = \frac{D}{2} N^2 k_L 10^{-9}\,\mathrm{H/cm}. \qquad (16)$$

The correction factor k_L is dependent on the ratio D/h, and can be taken from Figure 7.

The capacity C_2, which is composed of the inherent capacity C_2' of the secondary coil with electrode head and a following load capacitor C_B, must have the value

$$C_2 = \frac{1}{\omega^2 L_2} \tag{17}$$

so that the resonance conditions can be met with consideration of the requirement that $l = \lambda/4$. To a first approximation, for

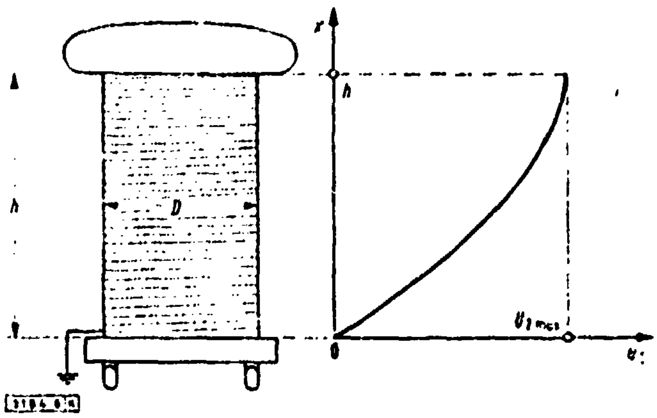

Figure 6. curve of the voltage compared to ground along the secondary coil of a Tesla transformer for the case $l = \lambda/4$, with l as the wire length of the secondary coil

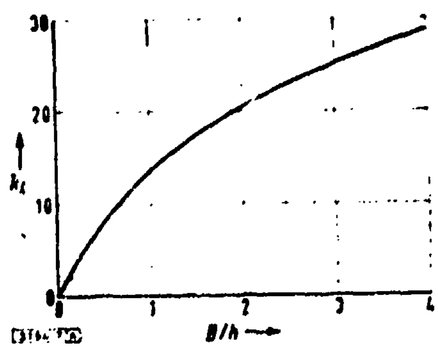

Figure 7. Correction factor k_L as a function of the ratio D/h of a single-layer cylindrical coil, for the calculation of the inductance according to formula (16)

C_2' can be used the capacity of the head electrode required to prevent premature discharges, with an addition of 30 to 50%. This value must, of course, be smaller by the amount $C_B = C_2 - C_2'$ than that calculated according to Equation (17). Assuming that the head electrode has the form of a round disc rounded off at circumferance, with a diameter d and a thickness $a \approx d/5$, the

standard value for its ground capacity as a function of the distance h from the ground and of the disc diameter d, can be taken from Figure 8.

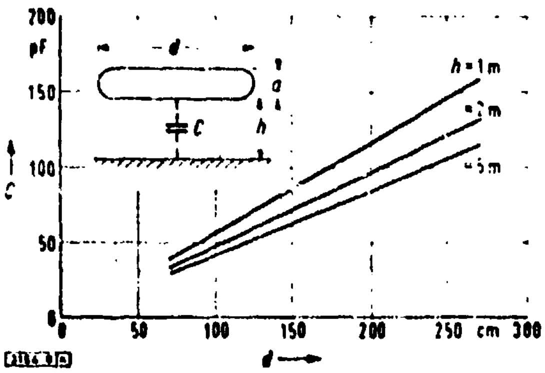

Figure 8. Standard values for the ground capacity C = f(d) of round discs with a diameter d and a distance from the ground of h, at a ratio of disc diameter d to disc thickness a of approximately 5.

It is apparent from Figure 9 that the lowest possible frequency f of a Tesla transformer with reasonable geometric dimensions depends to a great degree on the voltage for which it is designed, i.e., its structural height is of appropriate size. The [illegible] characterized range in Figure 9 results when the following assumptions, largely corresponding to reality, are made:

1. The required winding height h is 100 cm for each 400 kV of secondary voltage.
2. The greatest turn density is 2.5 turns per centimeter of coil height.
3. The ratio of coil height h to coil diameter D is in the range between 1 and 5.

Figure 9 shows that f_{min} increases with decreasing nominal voltage.

The attainable upper frequency f_{max} is essentially determined by the inherent C_2' of the secondary coil and the size of the added load capacitor C_B. However, it should be taken care that a reduction of the wire length l made necessary because $l = \lambda/4$ generally signifies a reduction of the inductance L_2 and therewith of the transformer ratio.

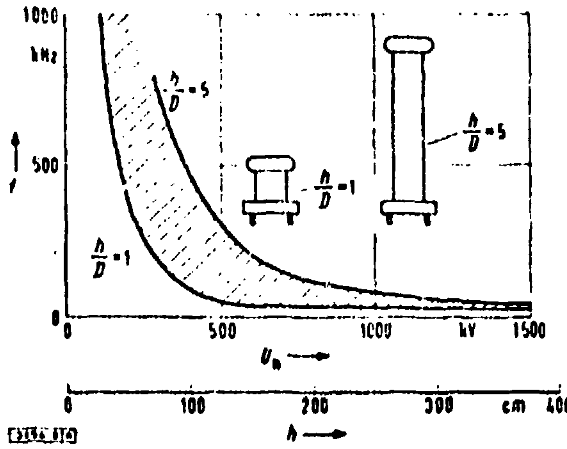

Figure 9. The dependence of the lowest operating frequency f_{min} of Tesla transformers on their nominal voltage U_n and their height h in the range $1 \leq h/D \leq 5$.

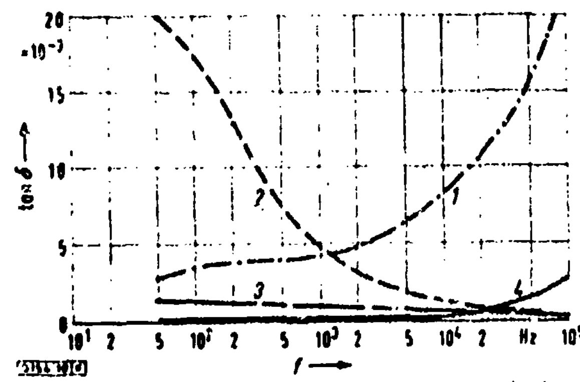

Figure 10. Loss factors of condensers with various dielectrics as a function of frequency.

1. Condenser with Clophene-paper dielectric.
2,3. Condensers with various ceramic dielectrics.
4. Condenser with Styroflex dielectric.

THE DAMPING OF TESLA TRANSFORMERS

In general, as small a damping as possible is desirable, in order to approach the theoretical transformer ratio of the transformer. As was explained in the preceding section, the voltage is composed of two individual oscillations with frequencies ω_I and ω_{II}, or f_I and f_{II}. If the logarithmic decrements of the primary and secondary circuits of the transformer are designated with δ_1 and δ_2, where

$$\delta_1 = \frac{R_1}{f_0 2 L_1} \quad \text{and} \quad \delta_2 = \frac{R_2}{f_0 2 L_2} \quad \text{ist,}$$

then for the decrements of the coupled oscillations,

$$\delta_{I,II} = \frac{\delta_1 + \delta_2}{2} \cdot \frac{\omega_{I,II}}{\omega_0}$$

or

$$\delta_{I,II} = \frac{\delta_1 + \delta_2}{2} \cdot \frac{1}{\sqrt{1 \mp k}} . \tag{18}$$

The coupled oscillation with the higher frequency is, accordingly, the more strongly damped. The decrements of the primary and secondary circuits are of equal effect on the damping of the resulting oscillation.

In order to be able to take the damping effect into consideration in a suitable way in the computation of a Tesla transformer, it is necessary to estimate the sizes of the dissipative resistors R_1 and R_2. These resistors are to be considered as equivalent values for all sources of loss which are in effect in the two circuits. The equivalent resistance R_1 of the primary circuit is composed of

1. the equivalent resistance R_1' which is calculated from the dielectric losses of the primary condenser C_1 as $R_1' = \tan\delta/(\omega C_1)$,
2. the ohmic resistance R" of the primary coil and of the connecting leads between L_1 and C_1, with consideration of the skin effect, and
3. the equivalent resistance of R"', which is determined by the losses in the switching element occurring in the switching process. If a spark gap is used, for example, then the resistance R"' is identical with the arc resistance, whose size can be in the range of approximately 10^{-2} to 1 Ω, depending on the current strength, the electrode separation, the electrode material, and other parameters.

The equivalent resistance R_2 in the normal case, is given sufficiently precisely by the ohmic resistance of the secondary coil, also taking into consideration the increase brought about

by the skin effect. Consideration of leakage losses on the secondary winding can generally be omitted if the highly effective plastics such as polyvinyl chloride or polyethylene which are available nowadays, are used as wire insulation and if care is taken that the secondary coil is so designed with respect to its external dimensions, that in itself it is free from electrical discharges. The last condition can be considered as met, if the secondary voltage based on a 100 cm coil height is not made larger than 400 kV, and if the terminal electrode at the highest potential with respect to ground is chosen in correspondance with the maximum voltage.

The earlier examples show that all of the values influencing the damping can be estimated with satisfactory confidence. A rather large uncertainty exists merely in the consideration of the arc resistance of the primary circuit spark gap. As already pointed out, this can lie in the range of approximately $10^{-2}\ \Omega$ to $1\ \Omega$, depending on how the mentioned influencing parameters are taken care of. For each individual case, therefore, preparatory measurements will have to be carried out, which give the precise data on the arc resistance to be expected. With currents of 1000 A and electrode distances of approximately 2 cm, with transverse ventilation of the arc (air velocity = speed of sound), resistances of several hundred milliohms are to be expected, as measurements already carried out have shown.

Finally, the frequency dependence of the loss factor tan δ of condensers with different dielectrics shown in Figure 10, should also be pointed out. The tan δ is, in fact, a direct measure of the equivalent resistance R_1' contributing to the damping. It is clear from Figure 10 that condensers with ceramic dielectric are best suited for Tesla transformers which operate at frequencies above 10 kHz.

A TESLA TRANSFORMER FOR A SECONDARY VOLTAGE OF 1.5 MV

Figure 11 shows a Tesla transformer for a maximum no-load voltage of approximately 1.5 MV. Laminated paper cylinders with an outside diameter of 1.6 m and 1.5 m were used as carriers for the primary and secondary windings. Their height is 4 m. The terminal of the secondary winding at its upper end consists of a ring-shaped aluminum electrode which is slotted radially at one position so that it does not act as a short circuit turn. The other circuit elements required for operation of the transformer can be seen in Figure 12, such as feed transformer, primary condenser, and spark gap. These elements are enclosed for safety reasons and for reasons of space-saving, in a concrete pit covered over with thick wood planks. The electrical connection between the transformer and the powering three-phase current generator, and between the primary condenser and the primary winding of the Tesla transformer, are made by cable . The overall circuit of the equipment can be seen in Figure 2. The charging of the primary condenser C_1 is carried out by a three-phase transformer with a power of 200 kVA and a voltage of 30 kV maximum. This transformer is powered by a two-phase loaded three-phase current generator with a power of 300 kVA and a maximum voltage of 500 V. The spark gap in the primary circuit consists of two cylindrical electrodes, adjustable in their mutual separation, with tungsten tips, which are ventilated by a strong flow of compressed air perpendicular to the axis of the arc. The number of firings per unit time is increased by this ventilation. At the greatest possible electrode distance, approximately three to four firings can be expected during a 50 Hz half wave. With smaller electrode separations and with full utilization of the voltage of the feed transformer, approximately 10-12 firings can be obtained for each half wave.

Table 2. Characteristics of the 1.5 mV Tesla transformer

a) Nr.	b) Kenngröße	c) Primärkreis	d) Sekundärkreis
1	Kapazität	$C_1 = 0{,}[illegible] \cdot 10^{-6}$ F	$C_2 = 1{,}35 \cdot 10^{-10}$ F
2	Induktivität	$L_1 = 103 \cdot 10^{-6}$ H	$L_2 = 0{,}45$ H
3	Gleichstromwiderstand	$R_1 = 23 \cdot 10^{-3}\ \Omega$	$R_2 = [illegible]9{,}5\ \Omega$
4	Resonanzfrequenz	$f_1 = 20{,}3$ kHz	$f_2 = 20{,}3$ kHz
5	Kopplungsfaktor	$k = \frac{M}{\sqrt{L_1 L_2}} = 0{,}18$	
6	Koppelfrequenz	$f_I = 22{,}4$ kHz $f_{II} = 18{,}7$ kHz	
7	resultierende Frequenz	$f = \frac{f_I + f_{II}}{2} = 20{,}6$ kHz	

a. No., b. characteristic, c. primary circuit, d. secondary circuit.

1. capacity, 2. inductance, 3. direct current resistance, 4. resonant frequency, 5. coupling factor, 6. coupling frequency 7. resulting frequency.

Figure 13 shows the Tesla transformer in operation with maximum possible excitation. The length of the discharge channels proceeding from the high voltage electrodes lies in the range between 7 m and 9 m. The picture was obtained by multiple exposures with exposure times of approximately 1 sec each. Table 2 contains the essential characteristics of the two coupled oscillating circuits of the 1.5 mV transformer. The numerical figures for the values 4 to 7 apply only to the unloaded transformer.

In Figures 14 to 18, the results of various studies carried out on the Tesla transformer are illustrated. They provide an idea of how a Tesla transformer behaves with external loads which are provided either by a connected test device or by discharges at the terminal electrodes of the transformer.

Curves 1 to 4 in Figure 14 show the transformer ratio u measured at various load capacities, based on the theoretical highest possible transformer ratio u_{th}, as a function of the size of the primary condenser C_1. As can be seen, as a matter of fact,

for each case of a capacitive load, a new maximum can be attained for the relative transformer ratio by changing C_1, but the value of the new maximum drops with increasing load. The reasons for these drops are of different types. Among others, the resonant frequency of the coupled system consistently becomes smaller as a result of the new balance by enlargements of C_1 and $C_2 = C_2' + C_B$, and the deviation therefore becomes larger and larger from the condition: wire length of the secondary winding = 1/4 of the wave length.

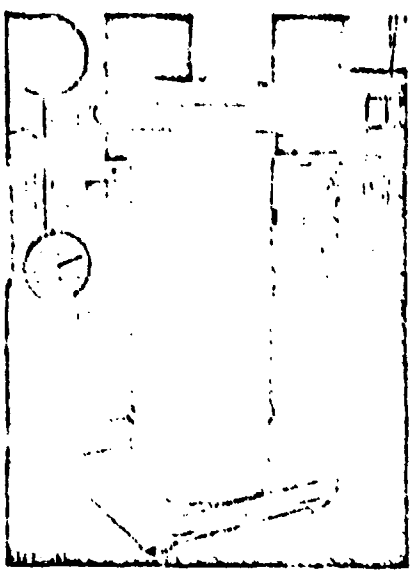

Figure 11. Tesla transformer for 1.5 MV.

With the use of the experimental points of Figure 14, two curves are plotted in Figure 15, which illustrate the dependence of the relative transformer ratio on the load capacity. Curve 1 applies to each

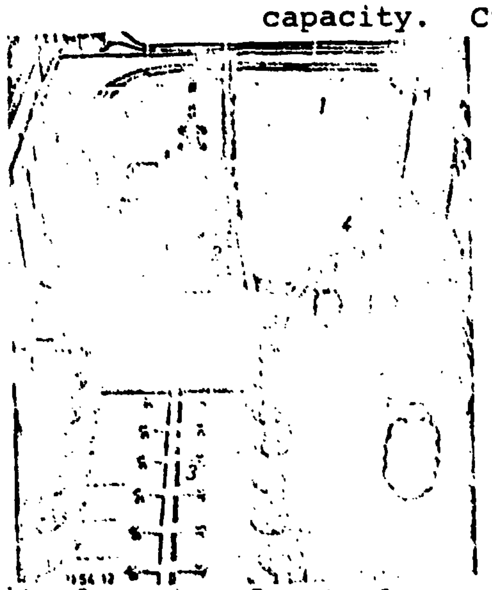

Figure 12. Circuit elements of a Tesla transformer other than the windings of the primary and secondary circuits shown in Fig.11
1. Feed cable from the three-phase current generator.
2. Feed transformer
3. Primary condenser
4. Spark gap ventilated with compressed air.
5. Cable terminal for connecting cable between spark gap and primary winding.

new matching of resonance by enlargement of C_1, Curve 2 applies to constant primary capacity $C_1 = 0.6\mu F$, i.e., for the transformer matched only to no-load, and then capacitatively loaded. Curves 1 and 2 show that the effect of a new matching increases with increasing capacity C_B.

Figure 13. Tesla transformer for 1.5 MV with full [ill.].

Figure 16 shows the dependence of the transformer ratio in the special case of the transformer tuned to resonance with no load and then loaded ohmically-capacitatively, on the size of the load resistance R_B parallel to ie secondary winding, or on the ratio R_B to ωL_2. The capacitative load caused by the ground capacities of the connecting lines to the resistor R_B and to the measuring device, remained unchanged. The measurement results indicate that the transformer is in a position, for example, at a voltage of 10^6 V, to produce a current with peak values of approximately 50 MA without significant voltage drop; this current would correspond to a load resistance of $2 \cdot 10^7 \Omega$..

Figure 17 shows the time curve of the primary voltage u_1 and of the secondary voltage u_2 of the described Tesla transformer, using a mechanical switch for closing the primary circuit. The beat-like curve of the voltage oscillations are recognizable both on the primary and on the secondary side. The time curve of the secondary voltage u_2 when a spark gap is used as the primary switch element, is seen in Figure 18. A beat cannot be formed here, since the arc between the electrodes of the circuit spark gap is extinguished after one half of a beat period, i.e., after reaching the maximum value of the secondary voltage. The secondary circuit therewith decays itself from this point in time onwards.

The large effect of the loss factor of the primary condensers on the damping, and with it the attainable transformer ratio, under some circumstances, should be again pointed out at this time. In measurements on the 1.5 MV Tesla transformer available, the use of condensers with ceramic dielectric or a plastic film (styroflex) dielectric was found to be outstandingly favorable. This becomes clear when the loss factors of condensers with this dielectric are compared with those of Clophene-paper condensers. Such a comparison has been made in Figure 10. It is seen there, that the ratio of tan δ of Clophene-paper condensers to the tan δ of ceramic or styroflex condensers at a frequency of 20 kHz, is approximately 10:1. For the series equivalent resistor R_1' of the primary condenser $C_1 = 0.6 \cdot 10^{-6}$F, there results at tan $\delta = 10 \cdot 10^{-3}$ at 20 kHz, a value of $133 \cdot 10^{-3} \Omega$. A comparison of the direct current resistance of $23 \cdot 10^{-3}$ Ω indicated in Table 2 for the primary side, with this equivalent resistance, shows the predominating influence of the high tan δ value. This also applies when it is considered that the ohmic resistance of the primary winding can increase as a result of the skin effect at 20 kHz to approximately twice the value of the direct current resistance.

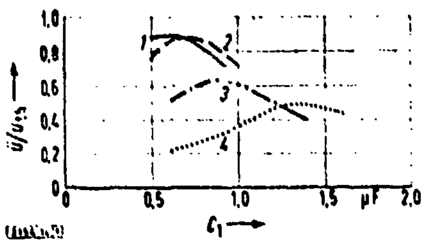

Figure 14. Dependence of the relative transformer ratio u/u_{th} on the size of the primary condenser C_1 at various sizes of capacitors C_B.

1 C_B = 8 pF
2 C_B = 35 pF
3 C_B = 85 pF
4 C_B = 200 pF

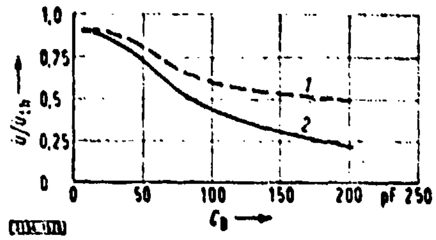

Figure 15. Dependence of the relative transformer ratio u/u_{th} on the size of the load condenser C_B.
1. Transformer for each case of load, tuned to resonance by enlargement of C_1.
2. Transformer tuned to no-load resonance. $C_1 = 0.6$ µF.

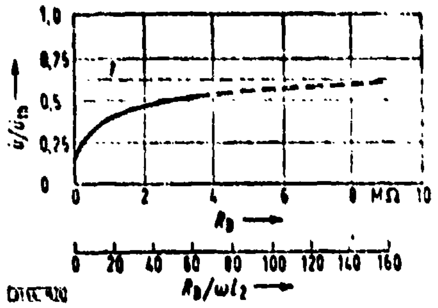

Figure 16. Dependence of the relative transformer ratio u/u_{th} on the size of the load resistor R_B, or on the ratio $R_B/\omega L_2$, with tuning to no-load resonance.
$C_1 = 0.6\ \mu F$.
1. Limiting value for $C_1 = 0.6 \mu F$ and $C_B = 65$ pF.

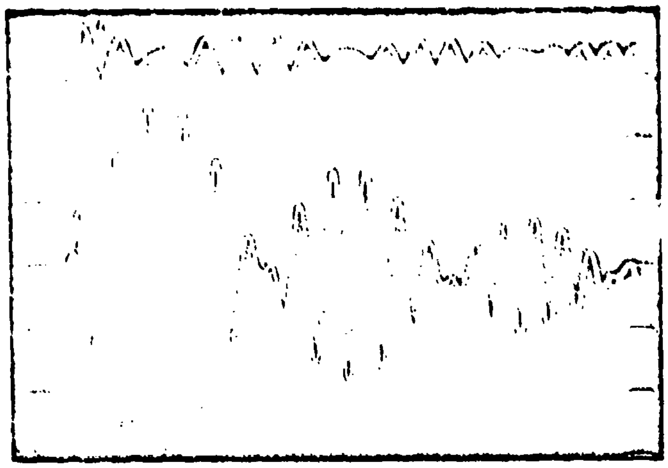

Figure 17. Time curve of the primary voltage $u_1 = f(t)$ and of the secondary voltage $u_2 = f(t)$ of a Tesla transformer for the case in which a mechanical switch is used as switching element.

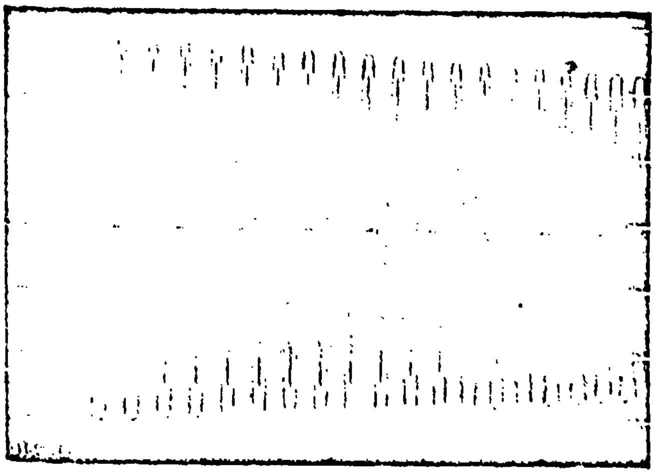

Figure 18. Time curve of the secondary voltage $u_2 = f(t)$ of a Tesla transformer for the case in which an air-ventilated spark gap is used as switching element. The voltage curve is visible only approximately from one fourth period before reaching the largest peak value of u_2.

In practice, the use of high frequency condensers for comparatively large values of capacity and for operating voltages in the order of magnitude of 10 to 30 kV, have limits for economic reasons. One must, therefore, be satisfied in the construction of Tesla instruments in general, with less expensive Chlophene-paper or oil-paper condensers, at the expense of the maximum obtainable transformer ratio.

SUMMARY

After general considerations of Tesla transformers, their theory is discussed. Among other matters, it is pointed out that the choice of the coupling factor k has an important effect on the maximum transformer ratio. The relationships between the nominal voltage and the frequency of transformers of this type are further discussed, which must be considered in proper design. Damping is discussed in a separate section. The various influencing parameters are listed, and it is particularly pointed out that the arc resistance of the spark gap generally used as the primary switching element, plays an important role. Finally, a Tesla transformer is described for a maximum no-load voltage of 1.5 MV, and measurement results are disclosed which are concerned chiefly with the effect of various types of loads on the transformer ratio.

BIBLIOGRAPHY

[1] *Drude, P.:* Über induktive Erregung zweier elektrischer Schwingungskreise mit Anwendung auf Perioden- und Dämpfungsmessung, Teslatransformatoren und drahtlose Telegraphie. Ann. Phys. Bd. 13 (1904) S. 512–561.

[2] *Drude, P.:* Rationelle Konstruktion von Teslatransformatoren. Ann. Phys. Bd. 16 (1905) S. 116–133.

[3] *Zenneck, J.:* Die Abnahme der Amplitude bei Kondensatorkreisen mit Funkenstrecke. Ann. Phys. Bd. 13 (1904) S. 822–826.

[4] *Kiebitz, F.:* Die vollständige Lösung der Differentialgleichungen zweier magnetisch gekoppelter, konstant gedämpfter elektrischer Schwingungskreise. Ann. Phys. Bd. 40 (1913) S. 138–156.

[5] *Bouwers, A.:* Elektrische Höchstspannungen. Verlag Julius Springer, Berlin 1939.

[6] *Hochhäusler, P.:* Der Teslatransformator als Hochfrequenzprüfgenerator und seine Untersuchung mit dem Kathodenoszillographen. Arch. Elektrotechn. Bd. 26 (1932) S. 518–534.

[7] *Rint, C.:* Handbuch für Hochfrequenz- und Elektro-Techniker. Verlag für Radio-Foto-Kinotechnik, Berlin 1953.

[8] *Heise, W.:* Tesla-Transformatoren für hohe Spannungen. AEG-Mitt Bd. 52 (1962) S. 354–361.

DISTRIBUTION LIST

DISTRIBUTION DIRECT TO RECIPIENT

ORGANIZATION		MICROFICHE
A205	DMATC	1
A210	DMAAC	2
B344	DIA/RDS-3C	9
C043	USAMIIA	1
C509	BALLISTIC RES LABS	1
C510	AIR MOBILITY R&D LAB/FIO	1
C513	PICATINNY ARSENAL	1
C535	AVIATION SYS COMD	1
C591	FSTC	5
C619	MIA REDSTONE	1
D008	NISC	1
H300	USAICE (USAREUR)	1
P005	DOE	1
P050	CIA/CRB/ADD/SD	1
NAVORDSTA (50L)		1
NASA/KSI		1
AFIT/LD		1
LLL/Code L-389		1

ORGANIZATION		MICROFICHE
E053	AF/INAKA	1
E017	AF/RDXTR-W	1
E403	AFSC/INA	1
E404	AEDC	1
E408	AFWL	1
E410	ADTC	1
E413	ESD	2
	FTD	
	CCN	1
	ASD/FTD/NIIS	3
	NIA/PHS	1
	NIIS	2